# SHIVAJI UNIVERSITY, KOLHAPUR

## CENTRE FOR DISTANCE EDUCATION

# Classical Mechanics

**(Mathematics)**

For

**M. Sc. Part-I**

**Paper (MT 105)**

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First Edition 2007
Second Edition 2010
Third Edition 2013
Fourth Edition 2019

Prescribed for **M. Sc. Part-I**

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Copies : 150

*Published by:*
**Dr. D. V. Muley**
Registrar,
Shivaji University,
Kolhapur-416 004

*Printed by :*

Superintendent,
Shivaji University Press,
Kolhapur-416 004

ISBN-978-81-8486-349-9

★ Further information about the Centre for Distance Education & Shivaji University may be obtained from the University Office at Vidyanagar, Kolhapur-416 004, India.

★ This material has been produced with the developmental grant from DEC-IGNOU, New Delhi.

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# Preface

It is hoped that students must learn mathematics not only to become a competent mathematicians but also skilled users of mathematics in the solution of problems in the real world. They must learn how to use their mathematical knowledge in solving the problems of the real world. I believe that through the study of classical mechanics, students will learn something about the art of applying mathematical knowledge to solve such problems. Comprehensive account of the mathematical artifact and numerous examples in this book will help the aspirants to develop an ability to use mathematics.

I have a great pleasure in presenting thoroughly a new book on classical mechanics in your hands. The material of the book is the standard post-graduate syllabus of most of the Indian Universities. This book “101 problems in Classical Mechanics” has been written for the use of students preparing for post-graduate examinations of Indian universities and SET/ NET aspirants. In such competitive examinations more emphases are given on examples. In order to fulfill the need of such students more than one hundred and ninety examples have been solved in the book, sometimes by alternative methods with complete mathematical techniques (theory) in the form of fifty three Theorems on five Chapters covered in this book. Another set of fifty examples with answers is given as an exercise in the Appendix. Efforts have been made to put the subject matter in as lucid and comprehensive manner as it is essential. Various reference books by the eminent authors have been utilized in the preparation of the text and the author is gratefully indebted to them. I have streamlined the examples and exposition, making the book easier to teach and learn from. It is hoped that the teachers, the students and large number of entrants to the competitive examinations will be benefited with the subject matter of this new book.

Any constructive suggestions for the improvement of the subject matter will be highly appreciated.

**L. N. Katkar**
Shivaji University, Kolhapur

**M. Sc. (Mathematics)**

# Classical Mechanics

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# Differential Equations

**(Mathematics)**

For

**M. Sc. Part-I**

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First Edition 2008
Second Edition 2010
Third Edition 2014
Fourth Edition 2015
Fifth Edition 2016
Sixth Edition 2019

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ISBN- 978-81-8486-012-2

★ Further information about the Centre for Distance Education & Shivaji University may be obtained from the University Office at Vidyanagar, Kolhapur-416 004, India.

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# Preface

Large numbers of students appear for M.A./M. Sc. Examinations externally every year. In view of this, Shivaji University has introduced the Distance Education Mode for external students from the year 2007-2008, and entrusted the task to us to prepare the Self Instructional Material (SIM) for aspirants.

It is hoped that students must learn Mathematics not only to become competent mathematicians but also skilled users of Mathematics in the solution of problems in the real world. They must learn how to use their Mathematical knowledge in solving the problems of the real world. Differential equations usually are description of physical systems. This book on Differential Equations consists of four chapters. Chapter one contains the complete discussion of linear equations with constant coefficients, including the uniqueness theorem. In chapter two linear equations with variable coefficients are trea. Equations with analytic coefficients are introduced and series solutions are obtained by a simple formal process. A detailed treatment of linear equations with regular singular points is discussed in chapter four. Classification of regular singular points and regular singular points at infinity is studied. In chapter five existence and uniqueness of solutions of first order initial value problem are established. The innumerable examples and exercises are given at the end of each unit.

The book introduces the students to some of the abstract topics that pervade modern analysis. The first chapter deals with the Riemann Stieltjes integration. The problems in Physics and Chemistry which involve mass distribution that are partly discrete and partly continuous can be solved by using Riemann Stietjes integrations. The Chapter 2 deals with convergence and uniform convergence of sequences of functions and series where as the Chapter 3 consists of multidimensional calculus. The Chapter 4 deals with implicit functions and extremum problems which have wide applications in optimization theory. Line integrals, surface integrals and Volume integrals are the subject matter of Chapter 5. This provides sufficient background to study the Gauss divergence Theorem and Stokes Theorem.

**Editor**

**M. Sc. (Mathematics)**

# Differential Equations

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## CENTRE FOR DISTANCE EDUCATION

# Advanced Calculus

**(Mathematics)**

For

**M. Sc. Part-I**

**Paper (MT 102)**

ISBN- 978-81-8486-512-7

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First Edition 2013

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# Preface

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This book introduces the student to some of the abstract concepts of advanced calculus.

The first unit deals with sequences of functions and uniform convergence. The Second unit discusses the properties which are preserved under uniform convergence. It also introduces the concept of double sequences. Unit three contains series of functions, convergence of series, subseries, double series and rearrangement theorems. The proof of Bernstein theorem, Abel's limit theorem, Tauber's theorem along with power series is discussed in unit four. Unit five deals with study of calculus of functions of several variables. In this unit the concepts of directional derivative and total derivative are introduced. The statements of inverse function theorem, implicit function theorem and the extrema of real valued functions of several variables along with examples are introduced in unit six. Concept of line integral along with

Green's theorem is discussed in the unit seven. Unit eight introduces the concept of surface integral along with curl and divergence of a vector field. The proof of stoke's theorem and Gauss divergence theorum is discussed in this unit.

We owe a deep sense of gratitude to the Vice-Chancellor Dr. N. J. Pawar who has given impetus to go ahead with ambitious projects like the present one. Thanks are also due to Mr. Sachin Kadam for computerizing the manuscript neatly and correctly. Any suggesions and corrections for improvement will be highly appreciated.

**Dr. H. G. Datar**
Willingdon College, Sangli

**M. Sc. (Mathematics)**

# Advanced Calculus

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ISBN-978-81-8486-080-1

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# Preface

This book in the form of "Notes of Algebra-I" is a natural outgrowth of the lectures delivered for M. Sc. Part-I students of Shivaji University. The primary purpose of this book is to facilitate the post graduate education in Algebra. The topics in the book will cover the syllabus of Algebra-I in detail for M. Sc. (Part-I) external students. For the basic ideas in Group theory and Ring theory students are advised to read in detail the other text books of Algebra.

First chapter deals with Group theory and it covers the following articles 1) Isomorphism theorems, 2) Soluable groups, 3) Series of Groups, 4) Sylow theorems.

The second Chapter is on Ring theory and it especially deals with polynomial rings.

In the third chapter we discuss Module theory, where modules are the generalization of vector spaces which students have studied in their B. Sc. course. The list of the articles in this chapter is as follows.

1) Modules 2) Sum and direct sum of submodules 3) Noetherian and Artenian Modules.

We owe a deep sense of gratitude to the Vice-Chancellor Dr. N. J. Pawar who has given impetus to go ahead with ambitious projects like the present one. Dr. L. N. Katkar, Head, Department of Mathematics, Shivaji University has to be profusely thanked for the ovation he has poured to prepare the SIM on Algebra. We also thank the Director of Distance Education Mode Prof. (Dr.) Arun Bhosale for their help and keen interest in completion of the SIM.

**Editor**

**M. Sc. (Mathematics)**

# Algebra-I

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1) Modules

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# SHIVAJI UNIVERSITY, KOLHAPUR

## CENTRE FOR DISTANCE EDUCATION

# Real Analysis

**(Mathematics)**

For

**M. Sc. Part-I**

**Paper (MT 103)**

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First Edition 2013
Second Edition 2014

Prescribed for **M. Sc. Part-I**

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Registrar,
Shivaji University,
Kolhapur-416 004

*Printed by :*

**Shri. B. P. Patil**
Superintendent,
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ISBN- 978-81-8486-506-6

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# Preface

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The book introduces the students to some of the abstract topics that pervade modern analysis. The first unit deals with the algebra of sets and Borel sets. Measure theory is the study of special type of set functions initiated by a French Mathematician Henri Lebesgue. Unit two to six deal with the Lebesgue measure theory and integration. Unit two deals with Lebesgue measure. Measurable functions and their properties are discussed in Unit three. Lebesgue introduced the concept of an integral called as Lebesgue integral, which generalizes the Rieman integration. The concept Lebesgue integration is introduced in Unit four. In this unit Lebesgue integral of bounded functions, Lebesgue integral of non-negative measurable functions and theorems on general Lebesgue integral are introduced. Unit five and six deals with differentiation and integration of monotone functions & functions of bounded variations. Unit seven introduces concept of absolute

continuity and fundamental theorem of integral calculus. Unit eight consists of Minkowski and Holders inequality, convergence, completeness and Riesz Representation Theorem.

We owe a deep sense of gratitude to the Vice-Chancellor Dr. N. J. Pawar who has given impetus to go ahead with ambitious projects like the present one. Thanks are also due to Mr. Sachin Kadam for computerizing the manuscript neatly and correctly. Any suggesions and corrections for improvement will be highly appreciated.

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**M. Sc. (Mathematics)**

# Real Analysis

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## CENTRE FOR DISTANCE EDUCATION

# Complex Analysis

**(Mathematics)**

For

**M. Sc.-I**

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First Edition 2009
Second Edition 2014
Third Edition 2015
Fourth Edition 2016
Fifty Edition 2019

Prescribed for **M. Sc. Part-I**

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# Preface

The Shivaji University, Kolhapur has established the Distance Education Centre for external students from the year 2007-08, with the goal that, those students who are not able to complete their studies regularly, due to unavoidable circumstances, they must be involved in the main stream by appearing externally. The centre is trying hard to provide notes to those aspirants by entrusting the task to experts in the subjects to prepare the Self Instructional Material (SIM). Today we are extremely happy to present a book on Complex Analysis for M. Sc. Mathematics students as SIM prepared by us. The SIM is prepared strictly according to syllabus and we hope that the exposition of the material in the book will meet the needs of all students.

This book introduces the students the most interesting and beautiful analysis viz. Complex Analysis. As a matter of fact Complex Analysis is a hard analysis, but it is truly a beautiful Analysis. The first topic is an introduction to Complex analysis. The second unit deals with Mobius transformations. The third unit introduces the reader to the notion of complex integration. Fundamental theorem of algebra and maximum modulus theorem are the results covered in the unit four. Unit five and six cover concept of winding number, Cauchy's integral theorem, Open mapping theorem and Goursat theorem. Laurent series development, Residue theorem with its application to evaluation of Real integrals, Rouche's theorem and Maximum Modulus theorem are the results contained in last two units.

**Editor**

**M. Sc. (Mathematics)**

# Complex Analysis

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## CENTRE FOR DISTANCE EDUCATION

# Differential Geometry

**(Mathematics)**

For

**M. Sc. Part-I**

**Paper (MT 205)**

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First Edition 2014
Second Edition 2014
Third Edition 2019

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ISBN-978-81-8486-529-5

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Geometry begins with curves, surfaces, and then manifold - the highest abstraction in Mathematics. In fact the analysis of geometry has spurred many a new mathematical development. Its relevance to physics underlies the importance of the subject. Euclid the pioneer of 300 B.C. "Euclid Geometry" had written on his entrance door "If you do not know Geometry, don't enter in my house", which aptly describes the importance and need of the subject in the curriculum. In order to fulfill the need of the entrants numbers of examples have been solved in the book some times by alternate methods with complete mathematical techniques. The material of the book has been presented in as lucid and comprehensive manner as it is essential. Hence it is easier to teach and learn from. It is also hoped that the book will be a good asset to SET/NET aspirants too. The innumerable examples and exercise are meant for enhancing the enjoyment of Differential Geometry. In fact it is said that "The best way to understand Mathematics is to get your hands dirty by solving problems."

We owe a deep sense of gratitude to the Vice-Chancellor Dr. N. J. Pawar who has given impetus to go ahead with ambitious projects like the present one. We also thank the Director Prof. A. R. Bhosale of Distance Education and his staff for their help and keen interest in completion of the SIM.

**Prof. L. N. Katkar**
**Dr. M. S. Bapat**

**M. Sc. (Mathematics)**

# Differential Geometry

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**(Mathematics)**

For

**M. Sc.-I Sem. II**

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ISBN-978-81-8486-134-1

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# Preface

Large number of students appear for M.A./M.Sc. Examinations externally every year. In view of this, Shivaji University has introduced the Distance Education mode for external students from the year 2007-08 and entrusted the task to us to prepare Self Instructional Material (SIM) for aspirants.

The book entitled 'General Typology' is a natural outcome of the lectures delivered in the Mathematics Department by Prof. Mrs. Y. S. Pawar, while teaching the course for M. Sc. I students. The main aim of this book is to present the subject General Topology in a very simple and easily understandable way. Plenty of examples and counter examples (with solutions) are provided for each concept of General Topology. The proofs of almost all theorems are given with minute details. The whole text is divided into 19 units to cover the syllabus. Each unit is then divided into three or four articles. At the end of each unit, we have provided a set of exercises which will be useful to test the student's comprehension and ability.

We owe a deep sense of gratitude to the Hon' Vice-Chancellor who has given impetus to go ahead with ambitious projects like the present one.

**Editor**

## M. Sc. (Mathematics)

# General Topology

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Dear Students

The SIM is simply a supporting material for the study of this paper. It is also advised to see the new syllabus 2013-14 and study the reference books & other related material for the detailed study of the paper.

# SHIVAJI UNIVERSITY, KOLHAPUR

## CENTRE FOR DISTANCE EDUCATION

# Linear Algebra

**(Mathematics)**

For

**M. Sc. Part-I**

**Paper (MT 201)**

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Shivaji University,
Kolhapur. (Maharashtra)
First Edition 2014
Second Edition 2014

Prescribed for **M. Sc. Part-I**

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Copies : 1,000

*Published by:*
**Dr. D. V. Muley**
Registrar,
Shivaji University,
Kolhapur-416 004

*Printed by :*

**Shri. B. P. Patil**
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ISBN-978-81-8486-528-8

★ Further information about the Centre for Distance Education & Shivaji University may be obtained from the University Office at Vidyanagar, Kolhapur-416 004, India.

★ This material has been produced with the developmental grant from DEC-IGNOU, New Delhi.

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# Preface

It is hoped that students must learn mathematics not only to become a competent mathematicians but also skilled users of mathematics in the solution of problems in the real world especially in Engineering. They must learn how to use their mathematical knowledge in solving the problems of the real world. I believe that through the study of Linear Algebra, students will learn something about the art of applying mathematical knowledge to solve such problems. Comprehensive account of the mathematical artifact and numerous examples in this book will help the aspirants to develop an ability to use Linear Algebra.

I have a great pleasure in presenting SIM on Linear Algebra in your hands. The material of the book is the standard post-graduate syllabus of most of the Indian Universities. In this book "Linear Algebra" has been written for the use of students preparing for post-graduate examinations of Indian universities and SET/ NET aspirants. In such competitive examinations more emphasis is given on examples. Efforts have been made to put the subject matter in lucid and comprehensive manner. Various reference books by the eminent authors have been utilized in the preparation of the text and the author is gratefully indebted to them. I have streamlined the examples and exposition, making the book easier to learn oneself. It is hoped that the teachers, the students and large number of entrants to the competitive examinations will be benefited with the matter of this book.

Any constructive suggestions for the improvement of the subject matter will be highly appreciated.

**Prof. Y. S. Pawar**
Shivaji University, Kolhapur

**M. Sc. (Mathematics)**

# Linear Algebra

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Each Unit begins with the section Objectives -

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# SHIVAJI UNIVERSITY, KOLHAPUR

## CENTRE FOR DISTANCE EDUCATION

# Numerical Analysis

**(Mathematics - MT-204)**

For

**M. Sc.-I**

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Shivaji University,
Kolhapur. (Maharashtra)
First Edition 2014
Second Edition 2015
Third Edition 2016
Fourth Edition 2019

Prescribed for **M. Sc. Part-I**

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Copies : 150

*Published by:*
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Registrar,
Shivaji University,
Kolhapur-416 004

*Printed by :*

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ISBN-978-81-8486-537-0

★ Further information about the Centre for Distance Education & Shivaji University may be obtained from the University Office at Vidyanagar, Kolhapur-416 004, India.

★ This material has been produced out of the Developmental Grant from UGC, Distance Education Bureau, New Delhi.

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# Preface

The Shivaji University, Kolhapur has established the Distance Education Centre for external students from the year 2007-08, with the goal that, those students who are not able to complete their studies regularly, due to unavoidable circumstances, they must be involved in the main stream by appearing externally. The centre is trying hard to provide notes to those aspirants by entrusting the task to experts in the subjects to prepare the Self Instructional Material (SIM). Today we are extremely happy to present a book on Numerical Analysis for M. Sc. Mathematics students as SIM prepared by us. The SIM is prepared strictly according to syllabus and we hope that the exposition of the material in the book will meet the needs of all students.

This book has grown from the lectures we deliver in the Department of Mathematics at Shivaji University, Kolhapur. The book is based on the curriculum recommended for M. Sc. Mathematics at Shivaji University, Kolhapur.

This book has four units. Unit 1 provides an introduction to error analysis and methods to estimate roots of polynomial and Transcendental equations. This unit deals with direct and iterative method for finding the roots of transcendentel and polynomial equations. In unit 2, the direct and iterative methods for the solution of a system of linear algebraic equations are discussed. The error analysis and convergence of iterative methods are also discussed. Various methods for finding eigenvalues and corresponding eigen vectors are explained. Unit 3 gives the numerical methods of differentiation and integration. Lagrange's interpolation and Newton's divided difference formula is derived that approximates a function by a polynomial of given degree. Uniqueness of interpolating polymial is proved. Error analysis for Lagvage's interpolation is carried out. Various methods for numerical differentiation and numerical integration are discussed along with their error analysis. Unit 4 deals with numerical solutions of ordinary differential Equations. Various methods used to determine the numerical solutions of ordinary differential Equations are discussed. Error analysis is for all the methods is given.

All the units are followed by solved problems. A good number of examples have been solved at the end of each unit to enable the student to understand the concepts described in the text. Good number of excercises are given at the end of each unit.

We hope that the content of the SIM will be helpful for the students having their education in distance mode.

**Editor**

**M. Sc. (Mathematics)**

# Numerical Analysis

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